

Instabilities in neutrino systems induced by interactions with scalars

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If there are scalar particles of small or moderate mass coupled very weakly to Dirac neutrinos, in a minimal way, then neutrino-anti-neutrino clouds of sufficient number density can experience an instability in which helicities are suddenly reversed. The predicted collective evolution is many orders of magnitude faster than given by cross-section calculations. The instabilities are the analogue of the “flavor-angle” instabilities (enabled by the Z exchange force) that may drive very rapid flavor exchange among the neutrinos that emerge from a supernova. These exchanges do require a tiny seed in addition to the scalar couplings, but the transition time is proportional to the negative of the logarithm of the seed strength, so that the size of this parameter is comparatively unimportant. For our actual estimates we use a tiny non-conservation of leptons; an alternative would be a neutrino magnetic moment in a small magnetic field. The possibility of a quantum fluctuation as a seed is also discussed. Operating in the mode of putting limits on the coupling constant of the scalar field, for the most minimal coupling scheme, with independent couplings to all three ν , we find a rough limit on the dimensionless coupling constant for a neutrino-flavor independent coupling of $G < 10^{-10}$, to avoid the effective number of light neutrinos in the early universe being essentially six. If, on the other hand, we wish to fine-tune the model to give a more modest excess (over three) in the effective neutrino number, as may be needed according to recent WMAP analyses, it is easy to do so.

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1. INTRODUCTION

In the last few years there has been an accumulating literature focused on the possibility that the neutral current $\nu - \nu$ interaction can induce rapid collective flavor exchanges of physical interest in situations in which there are initial flavor-energy correlations or flavor-angle correlations [1]-[14]. Up to now the application has been to the supernova neutrino pulse, where, apart from the dream that a galactic event will bring us information on neutrino properties directly from terrestrial detection, the flavor-energy correlations may matter to the explosion dynamics or to the possible R-process nucleosynthesis on the periphery of the supernova.

The purpose of the present paper is to show that the same kind of effect can enter in models in which neutrinos interact with scalar particles. Such interactions have been the subject of much speculation [15]-[25]. We believe that the exploration of different models of such an interaction will lead to a whole zoo of exotic phenomena, but we focus on the simplest example in this preliminary study, and we shall consider only one flavor of Dirac neutrino coupled to a single scalar field. The effects that we find are of most interest with respect to the early universe, in the period immediately prior to light element nucleosynthesis. In this system we have a thermal distribution of left-handed neutrinos, and their anti-particles, in nearly equal number. In standard theory a negligible number of right handed light neutrinos have been produced up to that early time, through a combination of collisions and the tiny neutrino mass term.

The collective effect that can come into play through the medium of the scalar particle exchange is the sud-

den and simultaneous creation of right-handed ν 's and left handed $\bar{\nu}$'s, keeping the lepton number equal to zero, and with a rate orders of magnitude greater than one would predict from a perturbative cross-section. The constraint on the model would thus be the need to avoid excessive production of right-handed ν 's and their antiparticles prior to neutrino decoupling in the big bang; that is, to avoid increase of the effective number of neutrino species to above the limits set by cosmology. In this system the collective effects of the $\nu - \nu$ force arising from Z_0 exchange vanish, owing to the cancellation between the particle and anti-particle terms in the effective potential.

As in the supernova case, the surrounding medium in our early-universe environment is filled with other particles than neutrinos. We shall consider only cases in which collision times for the neutrinos to scatter on these other particles are long compared to the instability times. Then the only effect of the other particles is through the constant background neutrino potential coming from the neutral current interactions with electrons and positrons (the interactions with nucleons being flavor independent and therefore irrelevant). In the early universe the number of electrons so nearly equals the number of positrons that this effect is negligible.

We first take a rather long detour through the existing lore of collective instabilities that derive from the Z_0 force. It is not our intention to write a review article, but we do need to discuss in detail what is the same and what is different in the scalar-mediated case. Furthermore, we wish to underscore two areas in which one might question the assumptions of the whole enterprise, including the work in refs. [1]-[14]: 1) the replacement of the in-

interaction Hamiltonian by a “forward” Hamiltonian; 2) after this replacement, and the extraction of equations of motion, a “mean field” assumption. In the case of potential issue #1, the issue is, in a pragmatic way, de-fused (but not put to rest) by staying in domains in which the interesting physics takes place over distances much less than a mean-free path as calculated from cross-sections. In the case of the mean-field assumption, we address the question directly from simulations with a finite number of neutrinos in a box. Assemblages of a few hundred neutrinos are computationally accessible on a desktop, and with this number we can support the mean-field method in some of the simpler coupling schemes. We return to these issues in section 4.

2. FORWARD HAMILTONIAN FROM Z_0 EXCHANGE.

In all cases we begin with an effective interaction Hamiltonian, H_I , arising from an exchange of a boson. Then if we are operating in, say, the MeV region of ν energies the $\nu - \nu$ interaction mediated by Z_0 exchange can be taken as a local four-Fermi interaction. In the case of our scalar exchanges we do want to cover the possibility of exchanged masses in the region below 1MeV and thus the propagator for the scalar particle contributes momentum dependence.

The key to all of the literature cited in refs.[1]-[14] is the replacement of H_I by a “forward” Hamiltonian. In this reduction we expand each field in plane wave modes; each term now is characterized by a momentum sequence, $\mathbf{p}_1, \mathbf{p}_2 \rightarrow \mathbf{p}_3, \mathbf{p}_4$. The forward interaction Hamiltonian is defined by omitting from the sum all terms except those in which $\mathbf{p}_1 = \mathbf{p}_3; \mathbf{p}_2 = \mathbf{p}_4$ or $\mathbf{p}_1 = \mathbf{p}_4; \mathbf{p}_2 = \mathbf{p}_3$. Then in discussing the evolution of some initial state under the influence of H_I we can ignore the kinetic term altogether. The only physical effects that ensue are now the exchange of flavors among momentum states (or we could equally say: the exchange of momenta among flavor states) in the case of the Z_0 model, or the alteration of helicities, with momenta unchanged, for the case of the scalar interactions. In dynamical calculations the free Hamiltonian H_0 , is now irrelevant; we deal entirely in a subspace of states of equal unperturbed energy.

These “forward” interaction Hamiltonians can now be completely expressed in terms of bilinear forms in neutrino creation or annihilation operators, $a_i^\dagger(\mathbf{p}) a_j(\mathbf{p})$, where $\{i, j\}$ indices are of flavor for the Z_0 case, and of helicity for the scalar case,

$$\begin{aligned} \rho_{i,j}(\mathbf{p}) &= a_i(\mathbf{p})^\dagger a_j(\mathbf{p}) \\ \bar{\rho}_{i,j}(\mathbf{p}) &= \bar{a}_j(\mathbf{p})^\dagger \bar{a}_i(\mathbf{p}), \end{aligned} \quad (1)$$

with the commutation relations,

$$[\rho_{i,j}(\mathbf{p}), \rho_{k,l}(\mathbf{p}')] = [\delta_{i,l} \rho_{k,j}(\mathbf{p}) - \delta_{j,k} \rho_{i,l}(\mathbf{p})] \delta_{\mathbf{p},\mathbf{p}'},$$

$$[\bar{\rho}_{i,j}(\mathbf{p}), \bar{\rho}_{k,l}(\mathbf{p}')] = [-\delta_{i,l} \bar{\rho}_{k,j}(\mathbf{p}) + \delta_{j,k} \bar{\rho}_{i,l}(\mathbf{p})] \delta_{\mathbf{p},\mathbf{p}'} \quad (2)$$

In terms of these operators the forward Hamiltonian is,

$$\begin{aligned} H_{\nu\nu}(\rho) &= \frac{\sqrt{2}G_F}{V} \sum_{\mathbf{p},\mathbf{q}} \sum_{i,j} [1 - \cos(\theta_{\mathbf{p},\mathbf{q}})] \\ &\times \left[\left(\rho_{i,j}(\mathbf{p}) - \bar{\rho}_{i,j}(\mathbf{p}) \right) \left(\rho_{j,i}(\mathbf{q}) - \bar{\rho}_{j,i}(\mathbf{q}) \right) \right. \\ &\left. + \left(\rho_{i,i}(\mathbf{p}) - \bar{\rho}_{i,i}(\mathbf{p}) \right) \left(\rho_{j,j}(\mathbf{q}) - \bar{\rho}_{j,j}(\mathbf{q}) \right) \right]. \end{aligned} \quad (3)$$

where the indices i, j range over the values e, μ, τ .

We divide the momentum space into N_B bins, each of them containing N_ν/N_B neutrinos, and distinguished by the superscript (α) . With few exceptions the instability literature is based on two-flavor models $i = \{1, 2\}$. In this case it is convenient to use angular momentum-like collective variables $S_0^{(\alpha)}, S_\pm^{(\alpha)}, S_3^{(\alpha)}$, formed from the $\rho(\mathbf{p})_{i,j}$ by summing over the region of momentum space, such that $\mathbf{p} \subset \alpha$, at the same time dividing out a factor N_ν/N_B .

$$\begin{aligned} S_+^{(\alpha)} &= N_B N_\nu^{-1} \sum_{\mathbf{p} \subset \alpha} b^\dagger(\mathbf{p}) a(\mathbf{p}), \quad S_-^{(\alpha)} = [S_+^{(\alpha)}]^\dagger \\ S_3^{(\alpha)} &= N_B N_\nu^{-1} \sum_{\mathbf{p} \subset \alpha} [b^\dagger(\mathbf{p}) b(\mathbf{p}) - a^\dagger(\mathbf{p}) a(\mathbf{p})] \\ S_0^{(\alpha)} &= N_B N_\nu^{-1} \sum_{\mathbf{p} \subset \alpha} [b^\dagger(\mathbf{p}) b(\mathbf{p}) + a^\dagger(\mathbf{p}) a(\mathbf{p})] \end{aligned} \quad (4)$$

where $a(\mathbf{p})$, $b(\mathbf{p})$ are the annihilation operators for the respective flavor states. We use a similar construction for anti-particle operators; but at this point we specialize to the case where none are present, for simplicity in sketching out methods. The anti-particles are easily reinstated for actual applications. The commutation rules for these collective operators are

$$\begin{aligned} [S_+^{(\alpha)}, S_-^{(\beta)}] &= N_B N_\nu^{-1} \delta_{\alpha,\beta} S_3^{(\alpha)} \\ [S_+^{(\alpha)}, S_3^{(\beta)}] &= -2 N_B N_\nu^{-1} \delta_{\alpha,\beta} S_+^{(\alpha)} \end{aligned} \quad (5)$$

etc.; within each beam, as for the Pauli matrices $\sigma_\pm = (\sigma_1 \pm i\sigma_2)/2$ and σ_3 , except for the factor of $N_B N_\nu^{-1}$. The Hamiltonian in terms of these variables for the case of the neutral current interactions is then,

$$\begin{aligned} H &= \sqrt{2} G_F n_\nu N_\nu N_B^{-1} \sum_{\alpha,\beta} \left[a_{\alpha,\beta} S_+^{(\alpha)} S_-^{(\beta)} + b_{\alpha,\beta} S_3^{(\alpha)} S_3^{(\beta)} \right. \\ &\quad \left. + c_{\alpha,\beta} S_0^{(\alpha)} S_0^{(\beta)} \right] + H.C. \end{aligned} \quad (6)$$

where n_ν is the neutrino number density and the matrices a and b are determined by the geometry of subdivisions into beams, taking into account the angular factors $(1 - \cos \theta_{\mathbf{p},\mathbf{q}})$. In the equations of motion that follow from (6) and (5), generically $idS/dt = [H, S]$, the factor of N_ν

from the Hamiltonian cancels the factor of N_ν^{-1} from the commutation rules and we obtain, e.g.,

$$i \frac{d}{dt} S_+^{(\beta)} = \sqrt{2} G_F n_\nu N_B^{-1} \sum_\alpha \left[a_{\alpha,\beta} (S_+^{(\alpha)} S_3^{(\beta)} + S_3^{(\beta)} S_+^{(\alpha)}) - b_{\alpha,\beta} (S_+^{(\beta)} S_3^{(\alpha)} + S_3^{(\alpha)} S_+^{(\beta)}) \right]. \quad (7)$$

The literature cited in refs.[1]-[11] is based on these equations, but with a further sweeping assumption, namely that on the right hand side we can replace the expectation values of products of S operators with the products of the individual expectation values, e. g.,

$$\langle S_+^{(\alpha)} S_-^{(\beta)} \rangle \rightarrow \langle S_+^{(\alpha)} \rangle \langle S_-^{(\beta)} \rangle, \quad (8)$$

generally characterized as the mean-field limit.

The diagonal elements of the matrices a and b are zero, due to the $(1 - \cos \theta)$ factor in the original Hamiltonian. Therefore there are no operator-order ambiguities on the right hand side of (7).

Clearly, in the mean field limit when we take pure flavor states as an initial condition we have $\langle S_+^\alpha(t=0) \rangle = 0$. Then the equations (7) and their coupled counterparts for the operators S_3^α say that nothing whatever happens. But if we took an initial values $\langle S_+^\alpha(0) \rangle \neq 0$, or added to H an ordinary neutrino oscillation term, then, according to the literature a bewildering number of things can happen that depend on the interplay of the two parts of the Hamiltonian. Beginning in sec. 4 we have studied some aspects of these effects perhaps more systematically than does the previous literature.

3. SCALAR PARTICLE EXCHANGE MODEL

We take the coupling to a scalar field ϕ to the Dirac neutrinos to be $H_I = g \bar{\psi} \psi \phi$. Now the effective Hamiltonian for the neutrino-neutrino interaction consists of a contribution from a scalar particle exchange and one from virtual annihilation.

We shall consider the scalar mass to be of the order of the neutrino energy scale or less. The \mathbf{p}, \mathbf{q} dependence in this case is more complex than in the Z_0 exchange case. For a each momentum \mathbf{p} there are now four states, neutrino and anti-neutrino each with either normal helicity ($-$ for ν , $+$ for $\bar{\nu}$) or opposite helicity. For the massless neutrino case, the coupling to the scalar connects the “normal” state only to the “opposite” helicity. However in constructing the forward Hamiltonian we shall need all 16 of the bilinear products of a creation and an annihilation operator, which we designate ρ_α . We use a notation in which $a(\mathbf{p})$ annihilates the ν state with (normal) negative helicity and $\bar{a}(\mathbf{p})$ the $\bar{\nu}$ state of (normal) positive helicity, with $b(\mathbf{p}), \bar{b}(\mathbf{p})$ annihilating the states of opposite helicity. The bilinears, $a^\dagger a, a^\dagger b, \bar{a}^\dagger b$ etc. have

an algebra

$$[\rho_\alpha(\mathbf{p}), \rho_\beta(\mathbf{p}')] = \sum_{\gamma=1}^{16} h_{\alpha,\beta}^\gamma \rho_\gamma(\mathbf{p}) \delta_{\mathbf{p},\mathbf{p}'} \quad (9)$$

The forward effective Hamiltonian for exchange of the scalar particle of mass m , written in terms of these operators, is,

$$H^{\text{eff}} = \frac{g}{V} \sum_{\mathbf{p}, \mathbf{q}} \sum_{\alpha, \beta} \left(\frac{p^\mu q_\mu}{|\mathbf{p}| |\mathbf{q}|} \right) \left(\xi^{\alpha, \beta} \rho_\alpha(\mathbf{p}) \rho_\beta(\mathbf{q}) \left[D_1(\mathbf{p}, \mathbf{q}) \right]^{-1} + \eta^{\alpha, \beta} \rho_\alpha(\mathbf{p}) \rho_\beta(\mathbf{q}) \left[D_2(\mathbf{p}, \mathbf{q}) \right]^{-1} \right), \quad (10)$$

where

$$D_1[\mathbf{p}, \mathbf{q}] = (\mathbf{p} - \mathbf{q})^2 + m^2 - (|\mathbf{p}| - |\mathbf{q}|)^2, \quad (11)$$

and

$$D_2[\mathbf{p}, \mathbf{q}] = (\mathbf{p} + \mathbf{q})^2 + m^2 - (|\mathbf{p}| + |\mathbf{q}|)^2. \quad (12)$$

Here the coefficients ξ and η are independent of the directions of \mathbf{p}, \mathbf{q} , as long as we use helicity states to define the $\rho_\alpha(\mathbf{p})$. The D_1 term comes from the exchange of the scalar between any pair of $\nu - \nu$ pair, $\bar{\nu} - \nu$ pair or $\bar{\nu} - \bar{\nu}$ pair.¹ The D_2 term comes from virtual annihilation of a $\bar{\nu} - \nu$ pair.

In contrast to the case of the Z_0 exchange terms of (3) that drove instabilities only in the case of nonisotropic momentum distributions, the form (10) will drive instabilities in perfectly isotropic systems of neutrinos and anti-neutrinos. These instabilities are the main point of the present paper.

We can then introduce *ab initio* angular averages of the operators in (10). However, also in contrast to the model of (3), we wish to consider a range of scalar particle mass m that ranges from very small, in comparison to the neutrino momenta, to equal or larger. Thus, while lacking the $\cos \mathbf{p}, \mathbf{q}$ factor that drives the angular dependences in the former case there is a continuum of $|\mathbf{p}|, |\mathbf{q}|$ values in the present case; for computational purposes we need to divide \mathbf{p} space into a number finite size bins, just as we did the angular space in the simulations reported in [14]. For our primary stability analysis we use only one bin, taking all neutrinos to be concentrated, say, at the peak of a thermal spectrum $|\mathbf{p}| = p_0$. We have examined simulations of the same problem with several bins and find no qualitative difference in the results for the instability conditions or results.

¹ Note that the spinor factor in the exchange amplitude is of the form $[\bar{u}_1(\mathbf{p}) u_2(\mathbf{q}) [\bar{u}_3(\mathbf{q}) u_4(\mathbf{p})]]$. We use a Fierz transform in reducing to the form shown in (10). We give the rather complex result, for the cases of interest, in sec. 6.

In the one-bin simulation the Hamiltonian is

$$H^{\text{eff}} = gn_\nu N_\nu \lambda_1 \sum_{\alpha,\beta} \xi^{\alpha,\beta} S_\alpha S_\beta + gn_\nu N_\nu \lambda_2 \sum_{\alpha,\beta} \eta^{\alpha,\beta} S_\alpha S_\beta, \quad (13)$$

where

$$\lambda_{1,2} = \int_{-1}^1 dx_{\mathbf{p},\mathbf{q}} D_{1,2}(p_0, p_0, x_{\mathbf{p},\mathbf{q}})^{-1} (1 - x_{\mathbf{p},\mathbf{q}}), \quad (14)$$

and

$$[S_\alpha, S_\beta] = N_\nu^{-1} \sum_\gamma h_{\alpha,\beta}^\gamma S_\gamma. \quad (15)$$

4. N NEUTRINO CALCULATIONS

In the mean-field (MF) reduction of equations (8) for the Z_0 exchange problem, or the analogous equations for the scalar exchange case, derived from (13) and (15), the particle number does not appear. We shall discuss solutions in the following sections. But insight is gained by solving the operator equations for systems with a finite number of neutrinos. This can be done numerically for numbers N_ν of a few hundred. One might have hoped that the rate corrections to the MF approximation would be of order N_ν^{-1} compared to the MF value. The situation turns out to be somewhat more complex, but the calculations reported below do support the MFA in the domains in which we need it. Additionally, the finite N_ν calculations show fascinating connections between unstable behaviors of the complete system with ν oscillation terms turned off, and unstable behaviors in the MF approximation when the oscillation terms are reinstated.

These calculations are based on the observation that the collective operators, $N_\nu S_\pm^{(\alpha)}, N_\nu S_3^{(\alpha)}$, which obey angular momentum rules, are sums of individual neutrino “spin” operators for the individual modes α . There are $2N_\nu$ basis states, but when the number of bins is small, the size of the problem can be much smaller. To begin with, the total “angular momentum” of each bin, $\mathbf{S}^{(\alpha)} \cdot \mathbf{S}^{(\alpha)}$, is conserved. Then if we have an initial state for the bin in which every “spin” is up, this “angular momentum” has maximum value, characterized by quantum number j_α . The basis of states for this one bin is thus effectively reduced from $2^{N^{(\alpha)}}$ to $N^{(\alpha)}$.

In the very simplest case Friedland and Lunardini [26] (see also [27]) found an analytic solution in the limit of a large number of ν 's for a two-flavor model with no ν oscillation terms. They take an initial state with two unidirectional beams of different flavors, at relative angle θ . The point is to calculate the time scale for the exchange of a macroscopic fraction of the flavors from one beam to the other. In the notation of the last section, we choose variables and S_\pm, S_3, T_\pm, T_3 , where the

S 's and T 's are the collective respective flavor matrices for the two beams. The operative neutral-current interaction is then the form

$$H = 2^{3/2} G N (S_+ T_- + S_- T_+ + \frac{a}{2} S_3 T_3), \quad (16)$$

where we have defined $G = 2^{-1/2} G_F n_\nu$. In the standard model case described by (3) the parameter a is unity; this is the case discussed in ref. [26]. But the $a < 1$ case, unphysical here, is a simple prototype both for Z exchange models cases with complex angular distributions, and for all the results for scalar exchange that we shall describe later.

We take the initial state to be an eigenstate of S_3, T_3 with eigenvalues $S_3 = 1$ and $T_3 = -1$; that is to say, the respective beams are of pure flavor and of different flavors. As displayed in (7) the equations of motion for these operators, when expressed in terms of neutrino densities, are free of any explicit occurrence of N_ν (or, equivalently, the volume). Nonetheless, in the products $S^{(\alpha)} S^{(\beta)}$ on the right hand side of (7) the operators act in the complete N_ν neutrino space, and the solutions will have N_ν dependence. Solving numerically for the case $a = 1$, the mixing time is found to be of order,

$$t_{\text{ev}} \sim G^{-1} N_\nu^{1/2}, \quad (17)$$

essentially the result of ref. [26] achieved by less elegant means.

There can be no physics here, since this mixing time for a system of a very large number of neutrinos would be longer than that originating in the ordinary cross-section. Thus the truncation to the forward Hamiltonian, as one might have expected, threw away almost all of the actual flavor exchanges predicted by the full theory.

In contrast, if $-1 < a < 1$, the results are very different. The analytic technique of Ref. [26] not being applicable here, direct numerical calculation for these cases gives the mixing time, which we now explicitly define as the time at which S_3 passes through the value zero, as

$$t_1 \sim C_a G^{-1} \log N_\nu \quad (18)$$

The coefficient C_a is of order unity for the fastest case $a = 0$. Eq.(18) holds with somewhat smaller coefficient for $|a| < .9$ in our numerical calculations. Again we are not at the moment suggesting a physical application of (18), though logarithms never really get large.

In Fig.1 we show results for values of $N_\nu = 100, 400, 1600$ for the two cases $a = .5$ and $a = 1$. The logarithmic dependence on N_ν in (18) is reflected in the equally spaced intercepts of the solid curves, and the $\sqrt{N_\nu}$ dependence in (17) in the behavior of the dashed curves.

We will not in this section try to analyze how these finite N results could reflect real behavior of neutrinos that are at large in space. In examples that we develop in the next section we find that, as one might even expect, when the basic model is augmented with a tiny seed

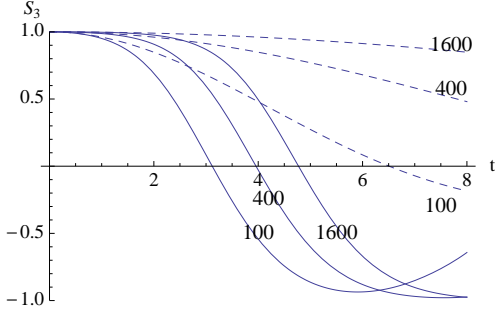


FIG. 1: Flavor exchange between two beams as a function of time. The curves are labeled by the number of neutrinos used in the simulation. The dashed curves are for the case $a = 1$ and the solid curves are for the case $a = .5$. The time is measured in units G^{-1} .

that can mix the states, the mixing time is given essentially by (18) but with $\log N_\nu$ replaced by $|\log \xi|$ where ξ measures the magnitude of the seed. For the case of Z_0 exchange and flavor mixing, the seeds are the ordinary oscillation parameters. For the scalar exchange model and helicity reversal, we return later to the question of possible seeds. If nature provided none (except for something so super-small that even the logarithm was huge), then possibly the $\log N_\nu$ model as is would be interesting. We would characterize the situation as “seeded by quantum fluctuations”.

5. MEAN FIELD CALCULATIONS

Turning to the mean field approximation, we add an ordinary neutrino oscillation term H_{osc} to the Hamiltonian (16),

$$H_{\text{osc}} = \epsilon N_\nu (S_+ + S_- + T_+/2 + T_-/2) \quad (19)$$

where we used different oscillation rates for the two different bins, in recognition of the fact that in applications the oscillation rates depend on energy, and the bins will typically have different energies. We use the commutation rules (5) among the S operators and their analogues among the T operators to obtain the equations of motion,

$$\begin{aligned} i \frac{d}{dt} S_+ &= G(S_3 T_+ - a T_3 S_+) + \epsilon S_3, \\ i \frac{d}{dt} T_+ &= G(T_3 S_+ - a S_3 T_+) + \epsilon T_3, \\ i \frac{d}{dt} S_3 &= -2G(S_- T_+ - S_+ T_-) + 2\epsilon(S_+ - S_-), \\ i \frac{d}{dt} T_3 &= -2G(T_- S_+ - T_+ S_-) + 2\epsilon(T_+ - T_-). \end{aligned} \quad (20)$$

With the mean field identities, $S_- = S_+^*$, $T_- = T_+^*$,

and the initial conditions

$$S_3(0) = 1, \quad T_3(0) = -1, \quad T_+(0) = T_-(0) = 0, \quad (21)$$

the equations (20) determine the evolution of the system. In the absence of oscillations, $\epsilon = 0$, none of the variables changes in time for any value of the parameter a .

In fig. 2 we plot the results of solutions of the set (20) for the two cases, for three different values of the vacuum oscillation parameter ϵ , with the range of the latter chosen to emphasize the similarities between fig. 2 and fig.1. Extending the calculation to the entire region

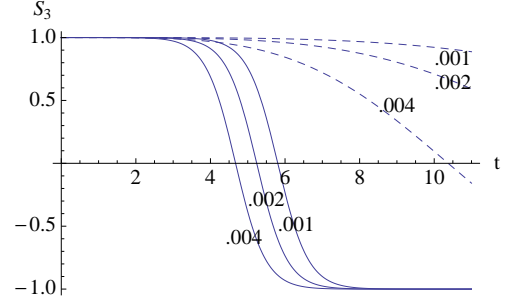


FIG. 2: As in fig.1, the flavor exchange against time, now from the mean field equations, and labeled by the strengths of the oscillation parameter ϵ/G , plotted for $a = .5$ (solid), and $a = 1$ (dashed)

$10^{-6} < \epsilon < .04$ we find a good fits to the values of the $S_3 = 0$ intercept t_2^{ev} to be given for the $a = .5$ data by

$$t_2^{\text{ev}} = G^{-1} \log[G/\epsilon] \quad (22)$$

and for the $a = 1$ data by,

$$t_3^{\text{ev}} = 1.85(G\epsilon)^{-.5}, \quad (23)$$

In spite of the intriguing similarities between fig.1 and fig.2, we still at this point have learned nothing about whether or not the mean-field equations give the complete solution in the limit $N_\nu \rightarrow \infty$. Ideally, now, we would go back and add the vacuum oscillation terms to the calculations of the last section and try to replicate the curves of fig. 2. The technical problem is the following: for the case with $\epsilon = 0$ and with our simple initial conditions there were only N_ν simultaneous coupled ODE's to be solved. When oscillations are included, there are N_ν^2 equations. Furthermore, as we see from comparing the $a = .5$ plots in figs. 1, and 2, a value $N = 100$ is not sufficient to push the transition region with $\epsilon = 0$ out far enough to see the specific effect of ϵ . Therefore our computational limit $N = 100$ for the case with the oscillation term included is insufficient for a test of the mean field approach for the values of ϵ used in fig. 2.

However, we have been able to squeeze in a simulation for a value $\epsilon = .2G$, in the $N_\nu = 100$ case, at times so small that the “finite N_ν ” contamination hardly enters (and so short that ordinary oscillation for the $G = 0$ case is negligible) and we find excellent agreement with the mean field result. We have also made a similar check of the MF approximation in a very limited domain in the $a = 1$ case.

The key to the vast qualitative difference in the behaviors for the $a = 1$ case and the $a < 1$ case for either of the above approaches is found by doing a standard instability analysis on the equations (20). Setting $\epsilon = 0$ and linearizing in the perturbations S_\pm, T_\pm we see from the last two of the set that, to linear order, S_3 and T_3 remain at their initial values. Then the solutions for S_\pm, T_\pm have simple exponential behavior $e^{\pm\lambda t}$ where $\lambda = \sqrt{(1-a^2)GN}$, and the real exponential for the case $a < 1$ signifies an exponential growth in response to a perturbation, such as the ϵ perturbation that gave rise to the rapid mixing in (22). The slower mixing in the case $a = 1$ of (23), is still much faster than the oscillation time ϵ^{-1} in the case that $G/\epsilon = G_F n_\nu / \epsilon \gg 1$, which holds in the regions of most interest in a supernova. In this case we can roughly say that two zero eigenvalues of the response matrix of the linearized system are effectively driven complex when ϵ is introduced and the system evolves a little, leading to a speed-up but not as dramatic a speed-up. We characterize this behavior as “moderately fast”, and that of (22) as “very fast”.

In the application to neutrino physics in a supernova, there have been a number of numerical simulations [1]–[11] that produce flavor exchange at a rate much faster than vacuum oscillation rates and occurring somewhat outside the neutrino-surface. These simulations must divide the neutrino flow into many bins, since the angular dependences coming from $\cos(\theta_{\mathbf{p},\mathbf{q}})$ in (3) are critical. The authors do not analyze their results in terms of the stability matrix of the linearized problem, but it is our conclusion that the closer-in of these phenomena are of our category, “moderately fast”, in the sense that they stem from a pair of zero eigenvalues. They also appear to fit the $(G_F n_\nu \epsilon)^{-.5}$ behavior of our simple case.

However, there cases in which the “very fast” evolution appears not only possible but likely [13], [14]. This can occur when we have multiple beams simulating an initial flavor dependent angular distribution more disorganized than those assumed in refs. [1]–[11] (which have perfect cylindrical symmetry). This appears to be the case in the region just below the surface of last scattering for a typical ν_e . In studies reported in [14] we started with a basic distribution in this region in which the ν_μ and ν_τ neutrinos (and anti-neutrinos) have angular distributions biased more outward than those of $\nu_e, \bar{\nu}_e$. With this distribution, we found “moderately fast” evolution, in a number of cases, with the most dramatic results for the “inverted hierarchy” assumption regarding the neutrino

mass matrix.

Then we added small random transverse distributions, breaking azimuthal symmetry. About 50% of the time the behavior shifted to that of very fast evolution, with mixing lengths much less than 1 cm. These results were robust with respect to introducing a neutrino energy spectrum, through the energy dependence of the oscillation terms, and the results were essentially the same for the normal and inverted hierarchies. In view of the turbulent fluid motion in this region, this suggested complete equalization of the energy spectra of the various neutrinos emerging from the neutrino-surface, with possible effects on explosion dynamics and the R process, and with certain effects on the neutrino pulse received on earth. In contrast to standard MSW transformations, which are driven completely by the change in background electron-density changes, these instabilities only require time to pass in order to establish the prerequisites for later sudden changes, with all model parameters remaining time independent.

6. SCALAR EXCHANGE MODELS

With the results of the previous sections in mind we now investigate the behavior of the model described in section 3. Beginning from the one-bin equations (13), (14) and (15), we note first that in the mean field interpretation we can, for the purposes of deriving the equations for the collective variables $S^{(\alpha)}$, cut down notation by eliminating all sums over modes \mathbf{p} in their definitions, and at the same time setting $N_\nu = 1$ in (13) and (15). We give a symbol for each of the 16 bilinear forms in the operators a, b, \bar{a}, \bar{b} , introduced in sec. 3, and then list all 120 independent relations of the commutator algebra. In this notation we have the 16 operators that we divide into three groups,

Group I

$$\begin{aligned} Q_3 &= b^\dagger b - a^\dagger a, \quad \bar{Q}_3 = \bar{b}^\dagger \bar{b} - \bar{a}^\dagger \bar{a}, \\ N_0 &= b^\dagger b + a^\dagger a + \bar{b}^\dagger \bar{b} + \bar{a}^\dagger \bar{a}, \\ Z &= a^\dagger a + b^\dagger b - \bar{a}^\dagger \bar{a} - \bar{b}^\dagger \bar{b}. \end{aligned} \quad (24)$$

Group II:

$$Q_+ = b^\dagger a, \quad Q_- = a^\dagger b, \quad \bar{Q}_+ = \bar{b}^\dagger \bar{a}, \quad \bar{Q}_- = \bar{a}^\dagger \bar{b} \quad (25)$$

Group III:

$$X_+ = \bar{b}^\dagger a, \quad X_- = a^\dagger \bar{b}, \quad Y_+ = b^\dagger \bar{a}, \quad Y_- = \bar{a}^\dagger b. \quad (26)$$

Group IV

$$V_+ = \bar{a}^\dagger a, \quad V_- = a^\dagger \bar{a}, \quad W_+ = b^\dagger \bar{b}, \quad W_- = \bar{b}^\dagger b. \quad (27)$$

The Hamiltonian (13) is given in terms of these operators by

$$H^{\text{eff}} = g n_\nu \lambda_1 \left[2(X_+ Y_+ + X_- Y_-) \right]$$

$$\begin{aligned}
& + (2 + \lambda)(V_+ W_+ + V_- W_-) + \lambda(X_+ X_- + Y_+ Y_-) \\
& + Q_+ \bar{Q}_+ + Q_- \bar{Q}_- - Q_3 \bar{Q}_3 / 2 + \bar{Q}_+ \bar{Q}_+ + \bar{Q}_- \bar{Q}_- \\
& - \bar{Q}_3 \bar{Q}_3 / 2 + \lambda(Q_+ \bar{Q}_- + Q_- \bar{Q}_+ - Q_3 \bar{Q}_3 / 2) + N_0^2 - \eta Z^2,
\end{aligned}$$

where $\lambda = \lambda_1 / \lambda_2$, $\eta = 1/4 + \lambda/8$ and,

$$\begin{aligned}
\lambda_1 &= (2p_0)^{-2} \left[1 + \frac{m^2}{4\mathbf{p}^2} \log \left[\frac{4\mathbf{p}^2 + m^2}{m^2} \right] \right] \\
\lambda_2 &= (2p_0)^{-2} \left[-1 + \frac{m^2}{4\mathbf{p}^2} \log \left[\frac{4\mathbf{p}^2 - m^2}{m^2} \right] \right]. \quad (29)
\end{aligned}$$

The equations of motion are given as usual by $id/dt S = [S, H]$ where we now use the commutation rules that come directly from the definitions (24)-(27).²

We shall always suppose that we start at $t = 0$ from a state in which all of the operators that mix one helicity with another, or that mix neutrino with anti-neutrino, are zero. The reason for dividing the operators into the four groups is that there are two cases in which the problem simplifies a little: a) when the mean fields of all of the operators of group II and group IV, namely $Q_\pm, \bar{Q}_\pm, V_\pm, W_\pm$ remain zero; or b) when the operators of group III and group IV $X_\pm, Y_\pm, V_\pm, W_\pm$, remain zero. In either case the linearized mean-field equations have a growing mode, when we take initial condition of all-active helicities. Both cases require a seed to make anything happen. In case b, we believe that a neutrino magnetic moment in a magnetic field can provide the seed, but discussing it in detail would require backtracking to put angle dependence in the equations and require binning in angle in the simulations. We treat case (a) in detail in the next section.

7. THE CASE; $Q_\pm = \bar{Q}_\pm = V_\pm = W_\pm = 0$

As in the earlier mean-field examples, nothing will happen until we add an analogue of the neutrino oscillation terms used above. In the present case we use the minimal lepton-number breaking bilinear, $H' = \frac{\epsilon}{m_\nu} \int d\mathbf{x} \psi(\mathbf{x}) C \psi(\mathbf{x})$, where m_ν is the neutrino mass and C the charge conjugation matrix. This translates, in our notation, into a mixing term in the forward Hamiltonian,

$$H' = \epsilon(X_+ + X_- + Y_+ + Y_-) \quad (30)$$

This will catalyze reactions in which there is sudden total exchange of helicity between the neutrinos and anti-neutrinos. Now defining $G = gn_\nu \lambda_1$, the effective Hamiltonian is,

$$\begin{aligned}
H &= G \left[2X_+ Y_+ + 2X_- Y_- + \lambda(X_+ X_- + Y_+ Y_-) \right. \\
&\quad \left. + 2Q_3 \bar{Q}_3 / 2 - \bar{Q}_3 \bar{Q}_3 / 2 - \lambda Q_3 \bar{Q}_3 / 2 - Z^2 \right] + H' \quad (31)
\end{aligned}$$

This, together with the commutation rules,

$$\begin{aligned}
[X_+, X_-] &= (Q_3 + \bar{Q}_3 - Z)/2, \\
[Y_+, Y_-] &= (Q_3 + \bar{Q}_3 + Z)/2, \\
[X_+, Q_3] &= [X_+, \bar{Q}_3] = -X_+, \\
[Y_-, Q_3] &= [Y_-, \bar{Q}_3] = Y_-, \\
[X_+, Z] &= 2X_+; [Y_-, Z] = 2Y_-, \quad (32)
\end{aligned}$$

determines the equations of evolution. We easily see that $Q_3 - \bar{Q}_3$ is conserved, and we will always take initial conditions such that $Q_3(t) = \bar{Q}_3(t)$. With that substitution (31) and (32) give,

$$\begin{aligned}
i \frac{d}{dt} X_+ &= G \left[2Q_3 [Y_- + (1 + \lambda)X_+] \right. \\
&\quad \left. - Z [Y_- + (4\eta + \frac{\lambda}{2})X_+] \right] + \epsilon(Q_3 - Z/2), \\
i \frac{d}{dt} Y_- &= G \left[2Q_3 [-X_+ - (1 + \lambda)Y_-] \right. \\
&\quad \left. - Z [X_+ + (4\eta + \frac{\lambda}{2})Y_-] \right] - \epsilon(Q_3 + Z/2), \\
i \frac{d}{dt} Q_3 &= 2G [X_+ Y_+ - X_- Y_-] \\
&\quad + \epsilon [X_+ - X_- + Y_+ - Y_-], \\
i \frac{d}{dt} Z &= \epsilon (-2X_+ + 2X_- + 2Y_+ - 2Y_-). \quad (33)
\end{aligned}$$

As in the earlier examples, we can investigate the possibility of very fast mixing by looking at the linear perturbations away from the trivial solution with $\epsilon = 0$, $X_+(t) = Y_-(t) = 0$, and $Q_3(t) = \text{const.}$ by looking at the two equations,

$$\begin{aligned}
i \frac{d}{dt} X_+ &= 2GQ_3 [Y_+ + (1 + \lambda)X_+], \\
i \frac{d}{dt} Y_- &= -2GQ_3 [X_+ + (1 + \lambda)Y_-]. \quad (34)
\end{aligned}$$

The solutions have time dependence $\exp[\pm \omega_0 t]$ where $\omega_0 = 2GQ_3 \sqrt{1 - (1 + \lambda)^2}$. If $-2 < \lambda < 0$ we thus find real exponential behavior that can drive rapid mixing. Taking a value of $\lambda = -1$ appropriate for most of our mass region and solving the full equations (33) we plot the results in fig.3 for a set of values of the ‘‘oscillation’’ parameter ϵ , ranging over eight decades. The intercepts with $Q_3 = 0$ are well fit by

$$t_{\text{mix}} = \omega_0^{-1} \log[\epsilon/\omega_0]. \quad (35)$$

² Before making the mean-field approximation, we could worry about the order of factors on the right hand side of the equations. But the commutator terms that distinguish one operator order from another would give corrections of relative order N_ν^{-1} in the end, and thus there is no ambiguity in the mean-field equations.

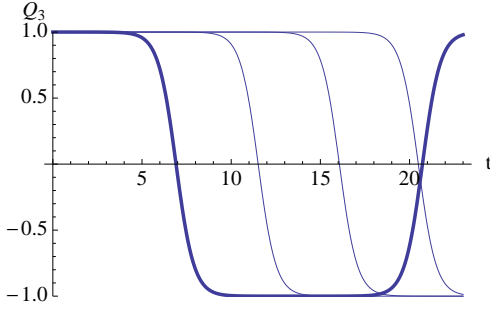


FIG. 3: Transformation of left handed ν 's and their antiparticles into right-handed counterparts, as catalyzed by a tiny lepton violating oscillation parameter ϵ . The results for $\epsilon/G = 10^{-3}$ are given by the heavy curve. Moving to the right, we show the plots for values, 10^{-5} , 10^{-7} and 10^{-9} . Time is measured in units of G^{-1} .

Although we have lepton number violation, measured by Z , catalyzed by the term (30), at no time in the evolution does Z become non-negligible, even for our largest value of ϵ .

Finally, we address the fact that the neutrino absolute momenta are to be distributed in a thermal spectrum, so that we must introduce multiple beams of neutrinos of different values $|\mathbf{p}|$, exactly as we introduced multiple angles in the Z_0 exchange model³. The effective Hamiltonian is now ,

$$\begin{aligned}
 H = GN_B^{-1} \sum_{\alpha, \beta}^{N_B} & \left(\lambda_1^{\alpha, \beta} \left[2X_+^{(\alpha)} Y_+^{(\beta)} + 2X_-^{(\alpha)} Y_-^{(\beta)} \right. \right. \\
 & \left. \left. - Q_3^{(\alpha)} Q_3^{(\beta)} / 2 - \bar{Q}_3^{(\alpha)} \bar{Q}_3^{(\beta)} / 2 - Z^{(\alpha)} Z^{(\beta)} \right] \right. \\
 & \left. + \lambda_2^{\alpha, \beta} \left[X_+^{(\alpha)} X_-^{(\beta)} + Y_+^{(\alpha)} Y_-^{(\beta)} - Q_3^{(\alpha)} \bar{Q}_3^{(\beta)} / 2 \right] \right) \\
 & + \epsilon \sum_{\alpha} \left[(X_+^{(\alpha)} + X_-^{(\alpha)}) \pm (Y_+^{(\alpha)} + Y_-^{(\alpha)}) \right]. \quad (36)
 \end{aligned}$$

Here the matrices λ_1, λ_2 are derived from $\int d[\cos \theta_{\mathbf{p}, \mathbf{q}}] D_{1,2}(\mathbf{p}, \mathbf{q})$ where the functions D are from (11) and (12), evaluated on a mesh of points $|\mathbf{p}|_i, |\mathbf{q}|_j$. In the numerical simulation we have taken a subdivision into N_B bins such that with an initial thermal spectrum every bin is equally occupied. The initial value of each $Q_3^{(i)}, \bar{Q}_3^{(i)}$ is then to be taken as $1/N_B$. If all the matrix elements of $\lambda_1^{i,j}$ are equal to each other and likewise for $\lambda_2^{i,j}$, then the results for the

time evolution of $\sum_i Q_3^{(i)}$ are the same as in the one bin model. But the scatter of values obtained in our procedure can be expected to have some effect. We have experimented in fitting the parameters $\lambda_1^{i,j}$ for the case of an initial thermal distribution at temperature T with up to 9 energy bins, each containing the same number of neutrinos, then solving the equations derived from (36) and comparing with the results of the single energy approximation, where we set the parameter $p_0 = 2T$. There is never more than a factor of two difference in the turn-over time, and discrepancies of this amount are found only in the region $10^{-5} < m/T < 10^{-4}$. We have not attempted the calculation in the region $m/T < 10^{-5}$.

Fig.4 shows an example of a multi-bin simulation in which the elements of the matrix $\lambda_1^{i,j}$ vary over a factor of 6, as compared to a single bin simulation in which λ_1 is taken as an average.

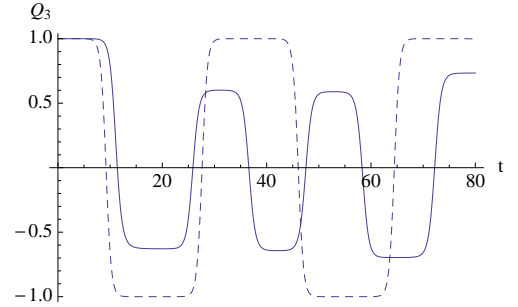


FIG. 4: The quantities in the same units as in fig. 4. The dashed curve is for the single bin theory as plotted in fig. 4, here shown for a mixing parameter $\epsilon/G = 10^{-4}$. The solid curve is for the multi-bin model with coupling constants varying over a factor of 6, rather than for a single G .

In all of the situations discussed above, our universe contained only neutrinos and antineutrinos. We believe, however, that the results are applicable under some conditions to real situations in which there are other particles that interact with neutrinos, such as the electrons and positrons in the early universe. A condition that we must demand is that the rate of neutrino reactions from ordinary scattering of neutrinos in the medium be small compared to the rate of transformation by the coherent mechanism. The limits on the scalar coupling that we shall quote demands that rate for the coherent process be at least five times that based on [(cross-section) \times density]. However, in the case of the scalar coupling model, there is a further issue that we must discuss before concluding that we do indeed have a model that predicts rapid creation of sterile helicity states. When we choose the scalar particle (ϕ) mass to be substantially less than the temperature, which we took as 1 MeV in the application, we must worry about the real process $\nu + \bar{\nu} \rightarrow \phi$;

³ Our angular averaging was legitimate, however; note that in the Z_0 exchange model we could have taken isotropic initial distributions and averaged away the $\cos(\theta_{\mathbf{p}, \mathbf{q}})$ factor in the interaction. But there the interesting physics is for non-isotropic distributions

for equal mixtures of active and sterile particles this reaction rate is exactly of the same order of magnitude as that of our rapid coherent process

In the initial state in our calculation, occupied only by active neutrinos and anti-neutrinos, this process is forbidden, but as our state with mixed in sterile particles develops, there would be reactions $\nu + \bar{\nu}_s \rightarrow \phi$ and $\bar{\nu} + \nu_s \rightarrow \phi$. One would guess that this would not change the qualitative results, but it would have been good to have an analytic treatment of the effect. The issue, moreover, appears to be essentially the same as that posed in ordinary neutrino oscillation theory when incoherent scattering is introduced. This has been discussed many times, going back at least to ref. [30], and we can state a typical outcome as follows: in the case of a large angle neutrino oscillation where the neutrino scatters as well, with flavor conservation but flavor dependence in the scattering rate, the scattering makes little difference over a few oscillation periods if the two time scales are comparable. If the scattering rate is much faster than the oscillation rate than the oscillation rate in the absence of scattering, then the flavor mixing is slowed, even frozen (in the limit of large scattering). But in our analogous system, the incoherent process of ϕ production is never strong enough to produce a major effect.

8. DISCUSSION

We have shown that the minimal Yukawa coupling of a Dirac neutrino to a scalar particle can, through a collective effect, lead to rapid production of otherwise nearly sterile right-handed neutrinos, in an initial thermalized state of left-handed neutrinos and anti-neutrinos. The inverse time scale is essentially of order $(g^2 T / \log[T^{-1}\epsilon])$, where ϵ measures strength of lepton-nonconservation from some other source. For any reasonable value of ϵ other than zero this is much faster than the rate of production $g^4 T$ in an ordinary perturbative calculation of $\nu_L + \bar{\nu}_L \rightarrow \nu_R + \bar{\nu}_R$ from cross-sections and density.

If we extend the model to include all three neutrino species, universally coupled to the scalar, then we can use the estimate (35) to put limits on the parameters g, ϵ to avoid complete population of the sterile (or “wrong-helicity”) states, contributing an additional three neutrinos to the mix in the period shortly before decoupling.⁴

The estimates do not depend critically on the mass of

the scalar particle as long as it is in the range $0 < m < T$. In setting limits on g we demand that the characteristic time for the helicity overturning effect be less than $1/5$ of the collision time for an ordinary reaction, like $e + \nu \rightarrow e + \nu$ in the medium at a temperature of 1 MeV . This gets us to a limit of about $g < 10^{-10}$, for values of the lepton number changing parameter $\epsilon > 10^{-15} eV$. This limit is less than those that come directly from particle experiment, and from supernova analysis, the latter at the $g < 10^{-4}$ level.

Alternatively, if, as seems to be indicated by recent WMAP analyses [28], more than three light neutrino species are needed, then it would be easy to adjust parameters to get any number between three and six in these models.

If couplings of the scalar are taken to change neutrino flavor, then the stability analysis gets very complex but is still straightforward. However the purpose of the present paper was to point out some range of possibilities rather than to try to do detailed phenomenology, with the secondary purpose to understand better the workings of the much discussed instabilities in the Z exchange case. In the latter case we have found that the mean-field assumption is supported by simulations involving a few hundred neutrinos.

In our detailed model, on which the above estimates are based, the seed was taken as a tiny term that violates lepton number conservation. It appears that a neutrino magnetic moment in a local magnetic field could also drive the effect. Given various more fundamental parameter assumptions, of a kind practiced in the game of “model building”, one might be able to argue that one or the other should dominate. A third possibility, briefly discussed at the end of sec.4 is that quantum fluctuations could serve as a seed.

Other physical systems can show collective behaviors analogous to the ones that we have discussed for neutrinos. Indeed an exactly analogous case has already been pointed out. Kotkin and Serbo showed [31], in a purely classical calculation, how a beam of polarized photons can exchange polarizations with another beam of polarized photons, through the (Heisenberg-Euler) quartic effective [32] interaction in QED. This conclusion does not require that the beams be monochromatic or coherent, though in practice the intensities needed for an experiment would surely require laser sources.⁵ As in the case of our very fast models, the calculated rate for the process

⁴ Of course, in the model we used in this paper the creation of the sterile states depletes the occupancy of the active states. But in the actual environment, before ν decoupling, the ordinary weak interaction rates, although slower than our transformation rate, will re-populate the active states; in the end we would expect rough thermal occupancy for all states.

⁵ The required parameters do not appear to be very far beyond those of the SLAC experiment [32] that detected the reaction $\gamma + \gamma \rightarrow e_+ + e_-$, rather misleadingly reported as “photon-photon” scattering. Although few would doubt the correctness or applicability of the effective $4\text{-}\gamma$ coupling for photons in the ultraviolet, its experimental confirmation would be of some interest.

is many, many orders of magnitudes faster than the rate for polarization exchange as calculated from the textbook photon-photon scattering cross-section, by virtue of being lower order in the effective coupling constant. A derivation and generalization of these results following the approach of the current paper can be found in ref [33]. This work was supported in part by NSF grant PHY-0455918.

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